[onecolumn]aa graphicx txfonts document Nonradial Oscillations of Neutron Stars with a Solid Crust Analysis in the Relativistic Cowling Approximation

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Nonradial oscillations of relativistic neutron stars with a solid crust are computed in the relativistic Cowling approximation, in which all metric perturbations are ignored. For the modal analysis, we employ three-component relativistic neutron star models with a solid crust, a fluid core, and a fluid ocean. As a measure for the relativistic effects on the oscillation modes, we calculate the relative frequency difference defined as $\delta\sigma/\sigma \equiv (\sigma_{GR} - \sigma_N)/\sigma_{GR}$, where σ_{GR} and σ_R are, respectively, the relativistic and the Newtonian oscillation frequencies. The relative difference $\delta\sigma/\sigma$ takes various values for different oscillation modes of the neutron star model, and the value of $\delta\sigma/\sigma$ for a given mode depends on the physical properties of the models. We find that $|\delta\sigma/\sigma|$ is less than ~ 0.1 for most of the oscillation modes we calculate, although there are a few exceptions such as the fundamental (nodeless) toroidal torsional modes in the crust, the surface gravity modes confined in the surface ocean, and the core gravity modes trapped in the fluid core. We also find that the modal properties, represented by the eigenfunctions, are not strongly affected by introducing general relativity. It is however shown that the mode characters of the two interfacial modes, associated with the core/crust and crust/ocean interfaces, have been interchanged between the two through an avoided crossing when we move from Newtonian dynamics to general relativistic dynamics. relativity — stars: neutron — stars: oscillations

Introduction

After the discovery of the r mode instability in neutron stars driven by gravitational radiation reaction (Andersson 1998; Friedman & Morsink 1998), nonradial oscillations of relativistic neutron stars has attracted much wider interest than before in astrophysics (see a recent review, e.g., by Andersson & Kokkotas 2001). The relativistic formulation of nonradial oscillations of *fluid* neutron stars was first given by Thorne and his collaborators (e.g., Thorne & Campolattaro 1967; Price & Thorne 1969; Thorne 1969a,b), and later on extended to the case of neutron stars with a solid crust in their interior (Schumaker & Thorne 1983, see also Finn 1990). Since one of the main concerns of these studies was gravitational waves generated by the stellar pulsations (e.g., Thorne 1969a), it was essential to include the perturbations in the metric $g_{\alpha\beta}$ in order to obtain a consistent description of the gravitational waves. However, it was the metric perturbations that made it extremely difficult to treat both analytically and numerically the oscillation modes in relativistic stars.

McDermott, van Horn, & Scholl (1983) introduced a relativistic version of the Cowling approximation, in which all the Eulerian metric perturbations $\delta g_{\alpha\beta}$ are neglected in the relativistic oscillation equations derived by Thorne & Campolattaro (1967). Under the relativistic Cowling approximation, they calculated the p-, f-, and g- modes of relativistic *fluid* neutron star models to examine how the oscillation modes depend on the model properties. The relativistic Cowling approximation employed by McDermott, van Horn, & Scholl (1983) was shown to be good enough to calculate the p-modes for non-rotating polytropic stars by Lindblom & Splinter (1990), who compared the fully relativistic oscillation frequencies to those obtained in the relativistic Cowling approximation. Assuming slow rotation, Yoshida & Kojima (1997) have also carried out similar computations for the f- and p-modes of polytropic models and they confirmed the good applicability of the relativistic Cowling approximation. Quite recently, Yoshida & Lee (2002) have shown in the relativistic Cowling approximation the existence of the relativistic r modesWe should note that lots of studies about the general relativistic r-mode have been done so far (e.g., Kojima 1998; Kojima & Hosonuma 2000; Lockitch, Andersson & Friedman 2001; Yoshida 2001; Ruoff & Kokkotas 2001).ith $l = m$, which are regarded as a counter part of the Newtonian r modes. We believe that the relativistic Cowling approximation is quite useful to investigate the oscillation modes of relativistic stars, although we understand that under the approximation we cannot discuss inherently relativistic oscillation modes like w-modes (see, e.g., Andersson, Kojima, & Kokkotas 1996).

f & Kokkotas 2001). w

In their Newtonian calculations, McDermott, van Horn, & Hansen (1988) have shown that cold neutron stars with a solid crust can support a rich variety of oscillation modes (see also Strohmayer 1991; Lee & Strohmayer 1996; Yoshida & Lee 2001). The purposes of this paper are to calculate the various oscillation modes of relativistic neutron stars with a solid crust in the relativistic Cowling approximation, and to discuss the effects of general relativity on the modal property of the oscillation modes. We regard this paper as an extension of the studies by McDermott, van Horn, & Scholl (1983) and McDermott, van Horn, & Hansen (1988). In §2, we derive relativistic oscillation equations for the solid crust in the relativistic Cowling approximation following the formulation developed by Schumaker & Thorne (1983) and Finn (1990). Numerical results are discussed in §3 for nonradial modes of three relativistic neutron star models with a solid crust. §4 is devoted to discussion and conclusion. In this paper, we use units in which $c = G = 1$, where c and G denote the velocity of light and the gravitational constant, respectively.

Formulation

For modal analysis of neutron stars with a solid crust, we solve general relativistic pulsation equations derived under the relativistic Cowling approximation, in which all the metric perturbations in the matter equations are neglected. We assume that the solid crust is in a strain-free state in the equilibrium unperturbed state and the strain in the crust are generated by small amplitude perturbations superposed on the unperturbed state (see, e.g., Aki & Richards 1980). For the background unperturbed state, it is therefore possible to assume a static and spherical symmetric state, for which the geometry is given by the following line element: eqnarray $(\mathrm{ds}^2)_{(0)} = g_{(0)\alpha\beta} dx^\alpha dx^\beta = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \,, metric$